

MMSE BASED SMP ALGORITHM FOR CHANNEL ESTIMATION IN MIMO SYSTEMS

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ABSTRACT

An iterative channel estimation algorithm based on Least Square Estimation (LSE), Minimum Mean Square Error (MMSE), Sparse Message Passing (SMP) for millimeter wave (Mm wave) MIMO system have now receiving tremendous interest in wireless communication systems. It is a spectral efficient system since it does not require prior knowledge of the channel distribution. It provide better performance since it can capture the inherent sparseness of millimeter wave channel. The channel matrix is considered as sparse hence it can detect the exact position and location of non zero entries in the channel matrix. In LSE-SMP, NMSE performance will lower with increasing iterations. LSE-SMP does not mitigate the noise effect while MMSE-SMP mitigate the noise effect. Also in LSE-SMP NMSE performance will decrease of sparsity ratio. Since millimeter wave channel is sparse, LSE-SMP will provide better performance for future wireless communication systems. MMSE-SMP can reduces the pathloss, blockage, shadowing effects and fading. When comparing all the methods conclude that MMSE-SMP will provide better performance b comparing the quantity NMSE and BER. That is NMSE is most lower for MMSE-SMP than LSE-SMP. It reaches the value almost 0.0001 while estimating the channel.

Keywords: *Sparse Message passing (SMP), Minimum Mean Square Estimation (MMSE), Millimeter wave, Iterative Channel Estimation.*

INTRODUCTION

MIMO(Multiple Input Multiple Output) has possessing large interest in future wireless communication systems. Multiple input multiple out technique is used to increase the data rate. In MIMO there are multiple transmit antennas and multiple receive antennas. Due to the use of antennas at both transmitter and receiver section a channel with dela and Doppler spread can get reduced [1] and tasks of channel tracking and ISI mitigation can be simplified. When a signal is transmitted from transmitter to receiver it have to pass through the channel. Hence, at the receiver side there is a need of estimating the channel. Several algorithms has proposed to estimate the channel. [2]–[4], [6]–[9]. They can be classified into three categories according to the required prior information of the channel. The algorithm in the first category requires the full knowledge of the channel's distribution, for example, the approximate message passing (AMP) algorithm [2] proposed by Donoho and Maleki. AMP is a low-complexity iterative Bayesian algorithm that can achieve approximately maximum a posteriori and minimum mean-squared error signal estimates. Iterative Detection/Estimation With Threshold (ITD-SE)

[3], Adaptive Compressed Sensing (ACS) estimation algorithm proposed in [4] and Orthogonal Matching Pursuit (OMP)[5] are classified into the second category, which needs partial prior information of the channel distribution, e.g., the degree of sparsity L.ITD-SE based Least Square Estimation (LSE) needs the fewer iterations, but its performance depends on the adaptive threshold selection scheme. ACS leverages the advanced compressed sensing theory and combines with the hybrid beam forming technique. Therefore, it is very suitable for the mm Wave systems. The algorithms in the third category do not need any prior knowledge of the channel distribution except the noise variance, e.g., Sparse Bayesian Learning (SBL) [6], LASSO[7], Expectation-maximization Bernoulli-Gaussian Approximate Message Passing (EM-BG-AMP) [8], etc. Recently, [9] proposed a modified mean field (MF) message passing-based algorithm, which also belongs to the third category and can deliver even better performance with the lower complexity than the conventional vector-form MFSBL algorithm by introducing a few hard constraint factors. This also provides a promising method for the future mm Wave channel estimation.

In this paper, we develop an iterative channel estimation algorithm based on the MMSE, Expectation-Maximization (EM) and Sparse Message Passing (SMP) for MIMO systems with large antenna arrays at both the transmitter and receiver.

We summarize our main contributions of this paper as follows.

- We formulate a sparse channel estimation problem and propose a novel sparse channel estimation algorithm. Compared with existing sparse channel estimation methods, ours can yield a better performance.
- We give the performance analysis of the proposed algorithm.
- We evaluate the performance of the proposed estimation algorithm. Numerical simulations show that our algorithm exhibits far better performance than the classical LSE and LSE-SMP estimator, as well as existing sparse channel estimators

2. SYSTEM MODEL

MIMO means multiple input multiple output. It is a method used for multiplying the capacity of radio link using multiple transmission and reception antennas. MIMO has become an important element in wireless communications[10]. It is also defined as a technique for sending and receiving more than one data signal over same radio channel.

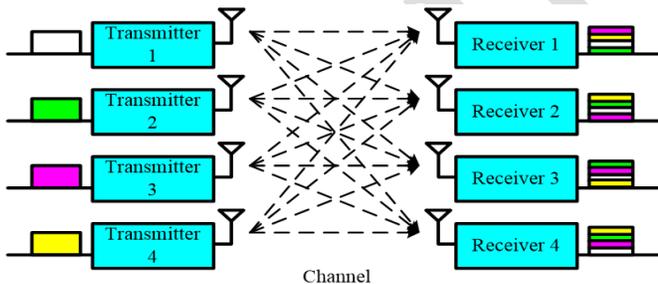


Fig 2.1 Simple MIMO model

A hybrid analog-digital mm Wave communication system that has N_t transmit and N_r receive antennas at the transmitter and receiver respectively, and both of them have N_{RF} RF chains. The transmitter and receiver communicate via N_s data streams, such that $N_s \leq N_{RF} \leq N_r$ and $N_s \leq N_{RF} \leq N_t$. Assuming frequency-flat fading channel, and that there is a $N_{RF} \times N_s$ base band precoder F_{BB} followed by an $N_t \times N_{RF}$ RF precoder F_{RF} in the downlink

transmission $F = F_{RF}F_{BB}$ as a $N_t \times N_s$ combined precoding matrix and C as $N_r \times N_s$ combining matrix, which is composed of the RF combiners C_{RF} and base band combiners C_{BB} . For the traditional hybrid analog-digital model, the observed signal at the receiver can be written as

$$y = C^H H F_s + C^H z \quad (1)$$

Where $H \in \mathbb{C}^{N_r \times N_t}$ is the channel matrix, $s \in \mathbb{C}^{N_s \times 1}$ is the transmitted signal, $y \in \mathbb{C}^{N_r \times 1}$ is the received signal and $z \in \mathbb{C}^{N_r \times 1}$ is the Gaussian noise with $z \sim \mathcal{N}(0, \sigma_n^2 I)$.

Since mm Wave channels are expected to have limited scattering, hence a geometric channel model with L scatterers is adopted. Each scatterer is further assumed to contribute a single propagation path between transmitters and receivers. Under this model, the channel H can be expressed as

$$H = \sqrt{\frac{N_r N_t}{\rho}} \sum_{l=1}^L \alpha_l a_r(\theta_l) a_t^H(\phi_l) \quad (2)$$

where ρ denotes the average path-loss between the transmitter and receiver, α_l is the gain of the l th path, $\phi_l \in [0, 2\pi]$ and $\theta_l \in [0, 2\pi]$ denote the l th path's azimuth angles of departure and arrival of the transmitter and receiver respectively. Finally, $a_t(\phi_l)$ and $a_r(\theta_l)$ are the antenna array response vectors at the transmitter and receiver respectively. If a uniform linear arrays is used, $a_t(\phi_l)$ can be written as

$$a_t(\phi_l) = \frac{1}{\sqrt{N_t}} [1, e^{j\frac{2\pi}{\lambda}d(\phi_l)}, \dots, e^{j\frac{2\pi}{\lambda}d(\phi_l)}]^T \quad (3)$$

where λ is the signal wavelength, and d is the distance between antenna elements.

For capturing the inherent sparse characteristic of the physical mm Wave modeling, a hybrid analog-digital communication architecture that is based on the beam space channel representation as shown in Fig.3.2.[12] This beam space representation also provides a tractable linear channel characterization, and offers a simple and transparent interpretation to the effects of scattering and array characteristics on channel capacity and diversity. Essentially, the beam space channel representation is to map the signal of the spatial domain to the signal of the beam domain by employing a carefully designed discrete lens antenna array instead of the electromagnetic antenna array.

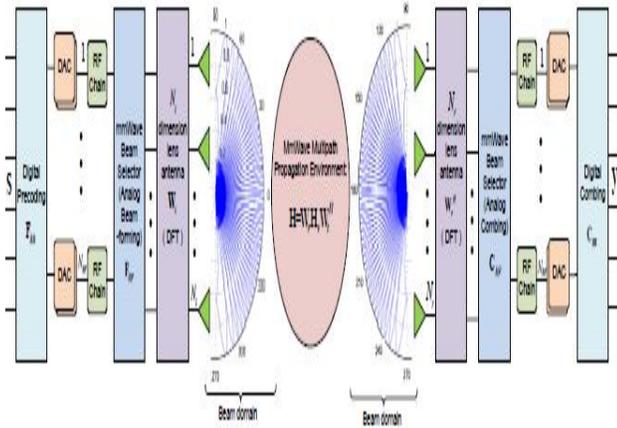


Fig 2.2 Beam space representation of MIMO model.

In particular, the lens acts as a virtual passive phase shifter, focusing the incident electromagnetic wave to a certain region. This lens, when used jointly with antennas, exhibits two significant properties: (i) focused signal power at the front end achieving high directivity and gain, and (ii) concentrated signal power directed to a sub-region of the antenna array. These properties make the lens a practical and energy efficiency tool for implementing the RF frontend in beam forming systems. The finite dimensionality of the signal space allows the beam space channel model that can be expressed as

$$H_v = W_r^H H W_t \tag{4}$$

Where $W_r \in \mathbb{C}^{N_r \times N_r}$ and $W_t \in \mathbb{C}^{N_t \times N_t}$ are channel invariant unitary DFT matrices.

3.SPARSE MESSAGE PASSING

An iterative channel estimation algorithm based on the MMSE and SMP algorithm as shown in Fig. 4.2, which is named MMSE-SMP. System model: it gives the structure of the MIMO model that is given in fig 3.2. MIMO means multiple antennas at the transmitter section and multiple antennas at the receiver section. Covariance matrix means that in probability theory and statistics, a covariance matrix, also known as auto-covariance matrix, dispersion matrix, variance matrix, or variance–covariance matrix, is a matrix whose element in the i, j position is the covariance between the i -th and j -th elements of a random vector. A random vector is a random variable with multiple dimensions. Each element of the vector is a scalar random variable. Each element has either a finite number of observed empirical values or a

finite or infinite number of potential values. The potential values are specified by a theoretical joint probability distribution. After designing the system model compute both the transmit side covariance matrix and receive side covariance matrix by taking the training sequences.

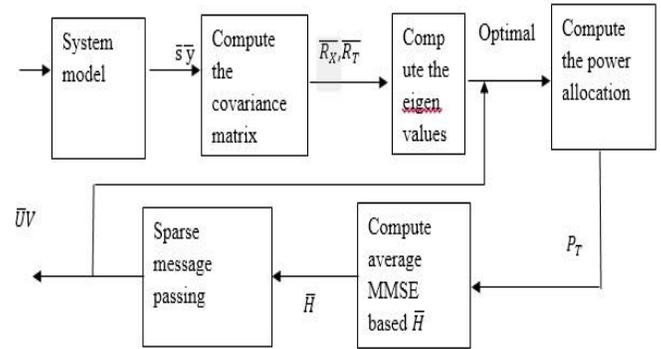


Fig. 4.2. The processing for the proposed MMSE-SMP algorithm.

Eigen value: a scalar associated with a given linear transformation of a vector space and having the property that there is some nonzero vector which when multiplied by the scalar is equal to the vector obtained by letting the transformation operate on the vector especially: a root of the characteristic equation of a matrix.

3.1 MMSE COARSE ESTIMATION

First make a system model using the following equation.

$$y(t) = Hx(t) + n(t) \tag{5}$$

Where $x(t) \in \mathbb{C}^{N_T}$ and $y(t) \in \mathbb{C}^{N_R}$ are the transmitted and received signals, respectively, and $n(t) \in \mathbb{C}^{N_R}$ represents arbitrarily correlated Gaussian disturbance. This disturbance models the sum of background noise and interference from adjacent communication links and is a stochastic process in .The channel is represented by $H \in \mathbb{C}^{N_R \times N_T}$. and the positive definite covariance matrix, $R \in \mathbb{C}^{N_T \times N_T}$ [13].

Then calculate the covariance matrix of the channel (R). Then calculate the eigen values. The channel with largest eigen value have larger power. According to this value of power, the channels are sorted largest power value is the channel having strongest signal. i.e in decreasing order. Then allocate the power. Signal is transmitted through largest power

channel, and the signal is received. The estimation is conditioned on the received signal from a known training sequence.

$$Y = HP + N \tag{6}$$

The received training signal of this system can be expressed as

$$\text{vec}(Y) = \tilde{P}\text{vec}(H) + \text{vec}(N) \tag{7}$$

Then, by pre-subtracting the mean disturbance $\text{vec}(N)$ from $\text{vec}(Y)$ can conclude MMSE estimator \hat{H}_{MMSE} of the Rician fading channel matrix H is

$$d = \text{vec}(Y) - \tilde{P}\text{vec}(\bar{H}) - \text{vec}(\bar{N}) \tag{8}$$

The error covariance C_{MMSE} is given by

$$C_{\text{MMSE}} = (R^{-1} + \tilde{P}^H S^{-1} \tilde{P})^{-1} \tag{9}$$

$$C_{\text{MMSE}} = R - R \tilde{P}^H (\tilde{P} R \tilde{P}^H + S)^{-1} \tilde{P} R \tag{10}$$

3.2 SPARSE MESSAGE PASSING ALGORITHM

After the Coarse Estimation of h_v , a fast iterative algorithm to find the positions of non-zero Entries is done. This algorithm is named sparse message passing since it can take full advantage of the channel sparsity and message passing algorithm. In order to get better performance, Firstly, decompose h_v into a diagonal coefficient matrix U_{h_v} and a column array b .

The column array $b = [b_{ij}]_{N_r \times N_t \times 1}$ ($i \in \{0, \dots, N_r\}, j \in \{0, \dots, N_t\}$) is called the position vector, and it represents the positions of non-zero in the coefficient matrix U_{h_v} . The $b_{ij} \in \{1, 0\}$ can be seen as a Bernoulli distribution. The SMP algorithm is considered for estimating positions of non-zero entries. It is similar to the belief propagation decoding process of the low density parity check code, in which the output message called extrinsic information on each edge is calculated by the messages on the other edges. Then, the h_v can be recast as

$$\begin{pmatrix} h_{11} & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & h_{N_r N_t} \end{pmatrix} \begin{pmatrix} b_{11} \\ b_{1N_t} \\ b_{N_r N_t} \end{pmatrix} \tag{11}$$

Then

$$\hat{h}_v = \tilde{S} h_v + \tilde{n} \tag{12}$$

3.4 UPDATE FOR MMSE ESTIMATION AND SPARSITY RATIO

A classical EM algorithm to learn the sparsity ratio. Since the channel vector h_v can be modeled as Bernoulli-Gaussian, then the EM update for

can be written as

$$\begin{aligned} &(\tau+1) \\ &= \arg \max_{\epsilon \in (0,1)} \sum_{j=1}^{N_t} \sum_{i=1}^{N_r} E \{ \ln p(h_{ij}; \hat{h} / (\tau) \bar{y}; \hat{\tau}(\tau)) \} \end{aligned}$$

4. RESULTS AND PERFORMANCE ANALYSIS

The results of a detailed numerical study on the performance of MMSE-SMP algorithm. For all numerical study, consider the channel estimation problem in a 32×64 mmWave MIMO system.

4.1 LSE-SMP ALGORITHM

4.1.1 Effect of Iterations

Fig. 5.1 shows the average channel estimation NMSE performance for the LSE-SMP algorithm under several turbo iterations. The result shows that the NMSE performance of the LSE-SMP algorithm will be lower with the increasing of iterations, and also find that the gap of the NMSE performance between the adjacent iterations for the LSE-SMP algorithm will be smaller with more iterations.

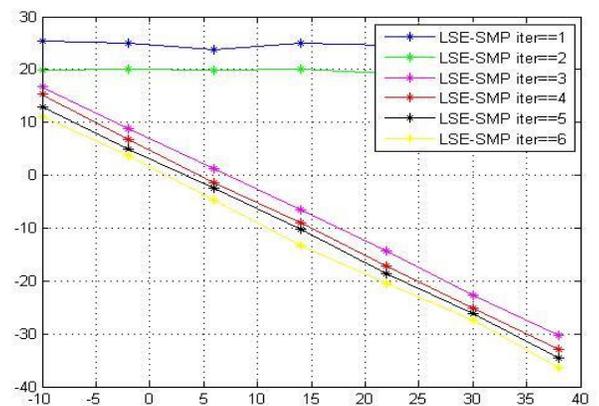


Fig 5.1 Simulation result that illustrate effect of iterations.

The main reason is that the parameters of sparsity ratio η and nonzeros position vector b are estimated more and more accurate. On the other hand, after the fifth turbo iteration, the NMSE performance have no significant improvement and it is very close to our analyzed LSE-SMP CRLB. This demonstrates that the convergence speed of the LSE-SMP algorithm is fast.

4.1.2 Effect of Sparsity Ratio

The figure illustrate to obtain its best performance and changed the sparsity ratio η from 0:007 to 0:80.

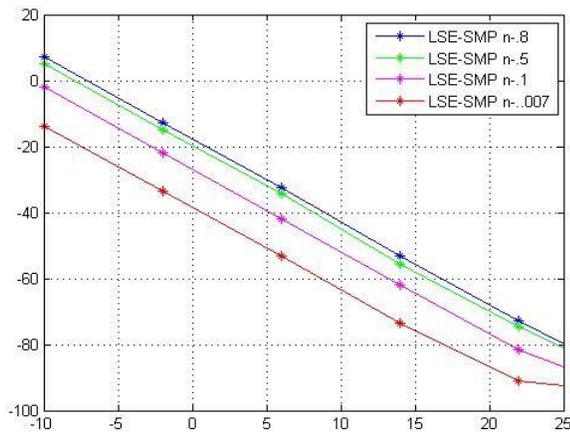


Fig 5.2 Simulation result that illustrate the effect of sparsity ratio

In addition, these results also show that the LSE-SMP is able to exploit the sparsity of the channel. To be specific, The NMSE performance of the LSE keeps unchanged under different sparsity ratios, while the NMSE of the LSE-SMP will decrease with the decrease of sparsity ratios. This indicates that the LSE-SMP scheme will be very suitable for channel estimation for mm wave systems since the channel of mm wave systems is sparse.

4.1.3 Complexity of the Algorithms

Figure 5.3 shows the comparison between the LSE and LSE-SMP algorithm and obtained their NMSE runtime. LSE-SMP is the best since it has lowest NMSE of 0.3816 as compared to LSE that having

NMSE 4.1054. Figure 5.4 shows the simulation result of comparison between LSE-SMP and MMSE-SMP and plotted the graph between SNR as input and NMSE as the output value. In LSE-SMP when SNR increases NMSE gets slightly decreased.

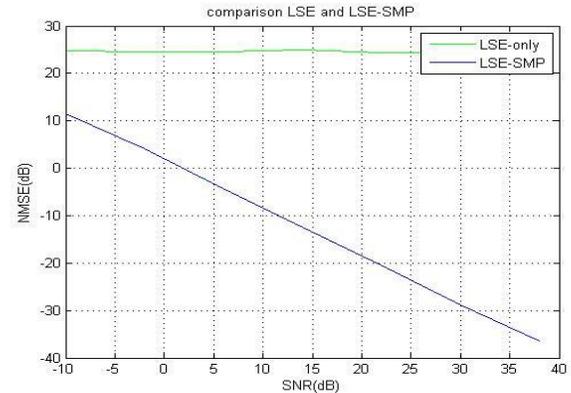


Fig 5.3 Simulation result that illustrate comparison of LSE and LSE-SMP

But in MMSE-SMP when SNR increases NMSE decreases. Hence concluding that the MMSE-SMP will provide better performance. And it can provide lower BER.

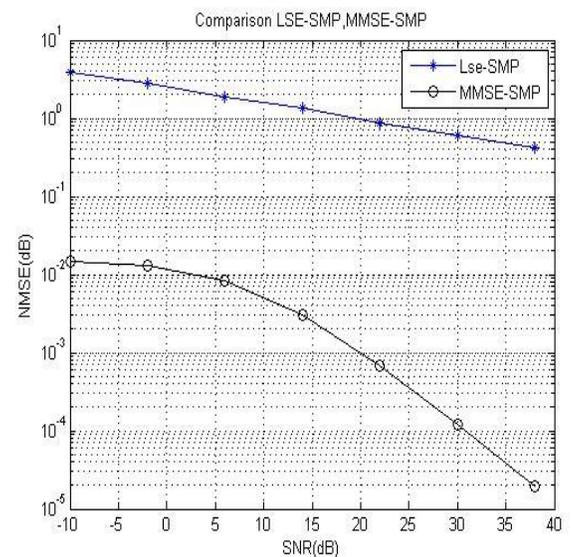


Fig 5.4 Simulation result that illustrate comparison of LSE-SMP and MMSE-SMP

4.2 MMSE-SMP ALGORITHM

4.2.1 Effect of Iterations

Figure 5.5 shows the average channel estimation NMSE performance for MMSE-SMP

algorithm under several turbo iterations. The results shows that the NMSE performance will lower with the increasing of number of iterations and find that the gap of NMSE performance between the adjacent iterations for MMSE-SMP algorithm will be smaller with more iterations

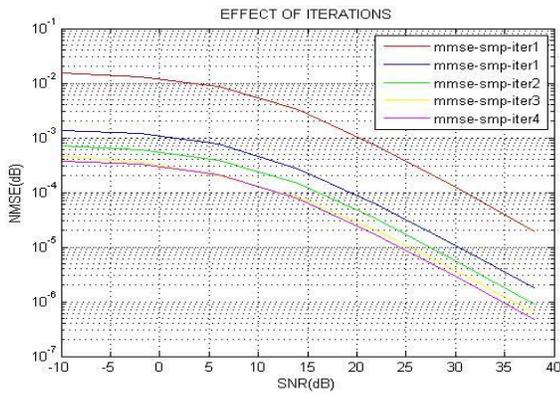


Fig 5.5 Simulation result that illustrate effect of iterations

On the other hand after fifth iterations, the NMSE performance have no significant improvement hence conclude that the convergence speed of MMSE-SMP is fast i.e , it just need only five iterations.

4.2.2 Effect of Sparsity Ratio

The effect of sparsity ratio to this algorithm is used to obtain its best performance and changed the sparsity ratio η from 0.007 to 0.80

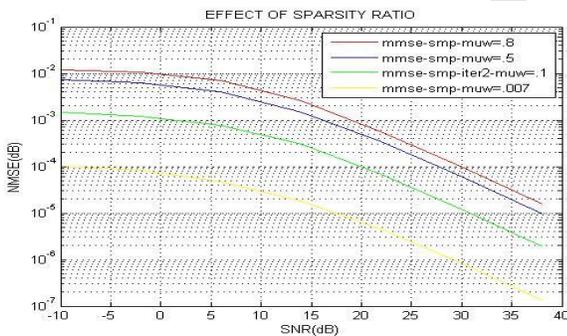


Fig 5.6 Simulation result that illustrate the effect of sparsity ratio

Simulation result in figure 5.6 Illustrate the effect of sparsity ratio. When the channel become more sparse, the performance of MMSE-SMP will be better. NMSE of MMSE-SMP will decrease with decrease of sparsity ratio. This indicates that MMSE-SMP scheme will be

very suitable for the channel estimation of mm wave systems. Since the channel of mm wave systems is sparse.

4.3 TABULAR EXPLANATION OF LSE-SMP ALGORITHM

Table 5.1 shows the various NMSE values of LSE-SMP algorithm under several turbo iteration and for various sparsity ratios. From the table NMSE value gets decreased as the number of iterations increases. Similarly effect of sparsity ratio also get decreased as the iterations increases.

Table 5.1 Various NMSE values of LSE-SMP

Effect of Iterations	NMSE	Effect of Sparsity Ratio	NMSE
1	24.5357	0.8	7.3300
2	19.4359	0.5	4.8370
3	-	0.1	-
4	29.9576	0.007	1.6272
5	32.5050		-13.23
	34.5747		

4.4 TABULAR EXPLANATION OF MMSE -SMP ALGORITHM

Table 5.2 shows the various NMSE values of MMSE-SMP algorithm under several turbo iteration and for various sparsity ratios. From the table 5.2 NMSE value gets decreased as the number of iterations increases. Similarly effect of sparsity ratio also get decreased as the iterations increases which illustrates better performance.

Table 5.2 Various NMSE values of MMSE-SMP

Effect of Iterations	NMSE	Effect of Sparsity Ratio	NMSE
1	0.0143	0.8	0.0115
2	0.0013	0.5	0.0074
3	0.009	0.1	0.0014
4	0.0006	0.007	0.0008
5	0.0005		

4.5 COMPARISON BETWEEN VARIOUS ALGORITHMS

Table 5.3 illustrates the comparison between the various algorithm which are used for estimating the channel. In LSE NMSE value is very high according to the given SNR value. Which is less when we compare it with LSE – SMP. For MMSE – SMP it is again very low. Which explains MMSE-SMP provide better performance. LSE have NMSE value is 4.1054 and it gets reduced to 0.3816 which shows that LSE-SMP is better than LSE. And again it gets reduced to 0.001 that shows MMSE-SMP is more better among the LSE and LSE-SMP.

Table 5.3 NMSE values of various algorithms

SI No	Methods	NMSE
1	LSE	4.1054
2	LSE-SMP	0.3816
3	MMSE-SMP	0.001

4.6 COMPARISON OF LSE-SMP AND MMSE-SMP IN TERMS OF BER

Table 5.4 illustrates the BER values of MMSE-SMP and LSE-SMP algorithms. In effect of iterations when comparing the MMSE-SMP BER value is low than LSE-SMP. In effect of sparsity ratio MMSE-SMP provide better performance since it provide lower BER.

Table 5.4 BER values of various algorithms

SINo	Properties	LSE-SMP	MMSE-SMP
1	Effect of Iterations	0.16	0.014
2	Effect of SparsityRatio	0.01	0.003

5. CONCLUSION

Mm wave has been possessing tremendous interest in future wireless communication due to its available band width of 30 GHz to 300 GHz. This millimeter wave possess many challenges like pathloss, fading, shadowing effects. To estimate the channel at the receiver end sparse channel estimation is used. A sparse channel estimation algorithm(LSE-SMP) and MMSE-SMP for mmWave MIMO systems, which leverages both virtues of the SMP, LSE and MMSE analyzed and showed that the algorithm is MVUE under

the assumption that the partial priori knowledge of the channel. Simulation results showed that the MMSE-SMP will provide better performance than compared to LSE and LSE-SMP algorithms, since all these algorithms are using for the channel estimation in MIMO systems. Since the BER value of MMSE-SMP is very low as compared to other two methods. Also, NMSE performance is also low when compared to LSE and LSE-SMP. This can be illustrated by simulating the result by using MATLAB code. From the Simulation results it is clear that MMSE estimator provides better performance than LS estimator in terms of mean square error (MSE) and Bit error rate (BER) whereas implementation of LS algorithm is much easier than MMSE algorithm.

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