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A Variable-Order Fractional Kelvin–Maxwell Hybrid Model for Enhanced Viscoelastic Creep and Relaxation Analysis

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Abstract

Accurate modeling of viscoelastic materials requires constitutive formulations capable of capturing long-memory and non-exponential time-dependent behavior. Classical integer-order models fail to represent such effects adequately, while constant-order fractional models, although improved, assume invariant memory characteristics. This study proposes a variable-order fractional Kelvin–Maxwell hybrid model to describe evolving viscoelastic behavior under creep and relaxation loading. The formulation integrates Caputo-type variable-order derivatives into a unified rheological framework and is implemented using a Grünwald–Letnikov numerical scheme. Validation is performed using a benchmark epoxy creep dataset. Comparative analysis demonstrates that the proposed model significantly reduces prediction error relative to classical and constant-order fractional models. The adaptive fractional order captures time-varying memory effects, providing enhanced physical interpretability and improved long-term deformation prediction. The proposed framework offers a computationally efficient and experimentally validated approach for advanced viscoelastic modeling.

Keywords: Variable-order fractional calculus; Viscoelastic materials; Creep modeling; Stress relaxation; Fractional Kelvin–Maxwell model; Memory effects; Numerical simulation

1. Introduction

Viscoelastic materials exhibit time-dependent mechanical behavior arising from the combined effects of elastic energy storage and viscous dissipation. Accurate prediction of creep, stress relaxation, and dynamic response is essential in engineering applications involving polymers, composites, biomaterials, and structural adhesives. Traditional rheological models based on combinations of springs and dashpots provide simplified representations of these phenomena; however, their integer-order differential formulations inherently predict exponential-type responses that do not adequately reflect experimentally observed power-law behavior [5-8].Fractional calculus has emerged as a powerful mathematical framework for modeling hereditary materials. By introducing non-integer order derivatives, fractional viscoelastic models naturally incorporate long-memory effects and provide superior agreement with experimental data using fewer parameters than classical multi-element models. Constant-order fractional Maxwell and Kelvin–Voigt formulations have demonstrated notable success in describing creep and relaxation behavior of polymeric systems.

Despite these advancements, most fractional viscoelastic models assume a constant memory index throughout the deformation process. Experimental observations suggest that material memory evolves due to microstructural rearrangement, progressive chain mobility, and internal relaxation mechanisms. A constant fractional order cannot fully capture this adaptive rheological behavior.To address this limitation, the present study proposes a variable-order fractional Kelvin–Maxwell hybrid model in which the fractional differentiation order evolves with time. This approach enables dynamic representation of memory effects within a unified constitutive framework. A robust numerical implementation based on the Grünwald–Letnikov scheme is developed, and the model is validated against benchmark epoxy creep data. The proposed framework provides enhanced physical interpretability and improved long-term deformation prediction, offering practical relevance for structural integrity assessment and reliability analysis of viscoelastic materials [14].

2. Literature Review

The mechanical behavior of viscoelastic materials has traditionally been modeled using combinations of elastic springs and viscous dashpots. Classical formulations such as the Maxwell and Kelvin–Voigt models provide simple representations of stress relaxation and creep phenomena, respectively. However, these integer-order models assume exponential time dependence and fail to capture the long-memory and power-law behavior observed in many polymeric and biological materials. Consequently, their predictive capability becomes limited, particularly for long-term deformation analysis[1], [3], [6], [7], [18].The introduction of fractional calculus into viscoelastic modeling provided a significant advancement in describing hereditary material behavior. Fractional derivatives inherently incorporate memory effects through power-law kernels, enabling accurate representation of non-exponential relaxation and creep responses. Early fractional models replaced classical dashpot elements with fractional elements characterized by derivatives of order 0<α<10 <α< 10<α<1, leading to improved agreement with experimental data while requiring fewer parameters than multi-element classical models [5], [6], [7], [8].

Constant-order fractional Maxwell and fractional Kelvin–Voigt models have been widely applied to polymers, asphalt materials, biomaterials, and composite systems. These formulations successfully describe intermediate and long-term viscoelastic behavior, particularly under creep and stress relaxation loading. Additionally, fractional models have been shown to provide better frequency-domain representation of storage and loss moduli compared to integer-order formulations [14], [15], [12], [19].Despite these improvements, most studies assumed a constant fractional order throughout the deformation process. Experimental observations, however, indicate that viscoelastic memory characteristics may evolve over time due to microstructural rearrangement, temperature variation, or progressive damage. Constant-order fractional models, although flexible, cannot fully capture such adaptive behavior [2], [4], [6].

To address this limitation, researchers began exploring variable-order fractional derivatives, in which the differentiation order becomes a function of time or state variables. Variable-order models offer enhanced adaptability and have shown promise in describing complex rheological responses. However, their application in unified Kelvin–Maxwell hybrid configurations remains relatively limited, particularly in the context of systematic numerical validation and experimental comparison [1], [2], [6].Therefore, there exists a need for a generalized viscoelastic framework that integrates classical rheological elements with variable-order fractional operators, supported by robust numerical implementation and experimental validation. The present study aims to address this gap by proposing and evaluating a variable-order fractional Kelvin–Maxwell hybrid model capable of capturing evolving memory effects in viscoelastic materials.

3. Mathematical Preliminaries

This section briefly presents the fundamental mathematical tools required for the development of the proposed variable-order fractional viscoelastic model. Emphasis is placed on the Caputo fractional derivative, variable-order fractional operators, Laplace transform formulation, and stability aspects relevant to viscoelastic constitutive modeling.

3.1 Caputo Fractional Derivative

Fractional calculus extends classical integer-order differentiation to non-integer (fractional) orders. Among several definitions available in literature, the Caputo derivative is widely adopted in viscoelastic modeling due to its compatibility with physically meaningful initial conditions. For a sufficiently smooth function $f(t)$, the Caputo fractional derivative of order $\alpha \in (0,1)$ is defined as:

$$D_t^{\alpha} f(t) = \frac{1}{\Gamma(1-\alpha)} \int_0^t \frac{f'(\tau)}{(t-\tau)^{\alpha}} d\tau, \qquad 0 < \alpha < 1 \qquad (1)$$

In this expression, $\Gamma(\cdot)$ denotes the Gamma function, which generalizes the factorial function to real and complex arguments. The term $f'(\tau)$represents the first derivative of the function with respect to time, and the kernel $(t-\tau)^{-\alpha}$introduces a power-law memory effect. The fractional order αcontrols the strength of this memory: smaller values of α correspond to stronger long-term memory, while values approaching unity recover classical first-order behavior.

Physically, the integral operator indicates that the present state of the system depends on the entire history of the function from time zero to the current time t. This non-local characteristic makes the

Caputo derivative particularly suitable for describing stress–strain relations in viscoelastic materials, where deformation history significantly influences present response.An important advantage of the Caputo derivative is that initial conditions can be prescribed in the classical form $f(0) = f_0$, which aligns naturally with experimental measurements of stress or strain at the initial time [2], [4], [6].

3.2 Variable-Order Fractional Derivative

Although constant-order fractional models successfully capture many viscoelastic behaviors, experimental observations indicate that the effective memory characteristics of materials may evolve due to structural rearrangement, aging, temperature variation, or loading intensity. To incorporate such adaptive memory behavior, the order of differentiation may be allowed to vary with time.The Caputo variable-order fractional derivative is expressed as

$$D_t^{\alpha(t)} f(t) = \frac{1}{\Gamma(1-\alpha(t))} \int_0^t \frac{f'(\tau)}{(t-\tau)^{\alpha(t)}} d\tau, \quad 0 < \alpha(t) < 1 \quad (2)$$

Here, $\alpha(t)$ is a continuous function of time representing the evolving memory index of the material. Unlike the constant-order case, the exponent of the kernel is now time-dependent, which allows the material response to transition between different rheological states during deformation.A commonly used representation for the time-varying fractional order is

$$\alpha(t) = \alpha_0 + \beta e^{-\lambda t} \quad (3)$$

Where α_0 denotes the long-term fractional order, β determines the initial deviation fromequilibrium, and λ is a positive parameter controlling the rate of evolution. This form enables modeling of materials that exhibit stronger memory effects at early stages and gradually stabilize over time.The introduction of variable order enhances modeling flexibility while maintaining mathematical consistency, if$\alpha(t)$ remains bounded within the interval $(0, 1)$[15], [12], [20].

5.3 Laplace Transform of the Caputo Derivative

The Laplace transform is an essential analytical tool in viscoelasticity, as it converts integraldifferential constitutive equations into algebraic equations in the complex frequency domain. For the Caputo fractional derivative of constant order $\alpha \in (0, 1)$,the Laplace transform is given by

$$\mathscr{L}\{D_t^{\alpha} f(t)\} = s^{\alpha} F(s) - s^{\alpha-1} f(0) \quad (4)$$

Where$F(s)$denotes the Laplace transform of $f(t)$, and s is the complex Laplace variable. The term s^{α} reflects the fractional power dependence in the frequency domain, while the second term incorporates the initial condition.This property is particularly advantageous in deriving stress relaxation and creep compliance functions. In fractional viscoelastic models, the relaxation modulus often takes the form

$$G(s) = \frac{E}{1+\tau^{\alpha} s^{\alpha}} \quad (5)$$

Where E represents the elastic modulus, and τ is a characteristic relaxation time. Inverse Laplace transformation of such expressions typically yields functions involving the Mittag–Leffler function, which generalizes the exponential decay observed in classical Maxwell-type models.For variable-order systems, closed-form Laplace solutions are generally not available, necessitating numerical inversion or direct time-domain discretization methods.

5.4 Stability Considerations

The introduction of fractional derivatives modifies the stability behavior of dynamical systems due to the inherent memory effect. Consider a linear fractional differential equation of the form

$$D_t^{\alpha} x(t) + a x(t) = 0, \quad 0 < \alpha < 1 \quad (6)$$

The system is asymptotically stable if the parameter a is positive. However, unlike classical first-order systems, the decay of solutions is not purely exponential; instead, it follows a power-law type attenuation governed by the fractional order α. Consequently, fractional systems exhibit slower relaxation and long-tail memory effects.

In variable-order systems, stability requires that the fractional order function $\alpha(t)$ remains bounded within the admissible interval and that material parameters such as elastic modulus and viscosity remain strictly positive. From a numerical perspective, stability also depends on the discretization scheme. For example, in Grünwald–Letnikov-type approximations, the time step must be selected carefully to ensure convergence and prevent numerical oscillations.These considerations are critical in constructing a physically consistent and computationally stable variable-order fractional viscoelastic model.

4. Proposed Variable-Order Fractional Kelvin–Maxwell Hybrid Model

In order to capture the complex time-dependent behavior of viscoelastic materials exhibiting evolving memory characteristics, a variable-order fractional Kelvin–Maxwell hybrid model is proposed. The formulation integrates the advantages of classical rheological elements with fractional operators while allowing the fractional order to vary with time.

4.1 Mechanical Configuration and Conceptual Framework

Classical viscoelastic behavior is commonly represented through combinations of springs and dashpots, corresponding respectively to purely elastic and purely viscous responses. The Maxwell model describes a spring and dashpot connected in series, effectively modeling stress relaxation but inadequately representing creep behavior over extended time periods. Conversely, the Kelvin–Voigt model, consisting of parallel spring and dashpot elements, captures creep behavior but fails to describe stress relaxation accurately.To overcome these limitations, a hybrid configuration is constructed by combining Maxwell and Kelvin–Voigt branches in a generalized fractional framework. In the present formulation, the viscous dashpot elements are replaced by fractional elements characterized by variable-order Caputo derivatives. This modification enables the model to incorporate long-memory effects and evolving rheological characteristics within a unified constitutive structure.The proposed configuration consists of:

- An elastic spring of modulus E_1,
- A fractional Maxwell branch defined by modulus E_2 and fractional viscosity parameterη_α
- A fractional Kelvin-type branch characterized by viscosity η_β

The fractional orders associated with the branches are allowed to evolve in time through a variable-order function$\alpha(t)$.Let $\sigma(t)$ denote stress and $\varepsilon(t)$denote strain. For the fractional Maxwell branch, the constitutive equation with variable order $\alpha(t)$may be expressed as

$$\sigma_M(t) + \frac{\eta_\alpha}{E_2} D_t^{\alpha(t)} \sigma_M(t) = \eta_\alpha D_t^{\alpha(t)} \varepsilon(t) \tag{7}$$

In this relation, η_α represents the generalized fractional viscosity parameter, and the operator $D_t^{\alpha(t)}$ introduces time-dependent memory. The ratio η_α / E_2 defines a generalized fractional relaxation time.For the fractional Kelvin-type branch, the constitutive equation may be written as:

$$\sigma_K(t) = E_1 \varepsilon(t) + \eta_\beta D_t^{\alpha(t)} \varepsilon(t) \tag{8}$$

WhereE_1 denotes the instantaneous elastic modulus, and η_β is the fractional damping parameter associated with creep resistance.Assuming parallel coupling of the branches, the total stress is obtained by superposition:

$$\sigma(t) = \sigma_M(t) + \sigma_K(t) \tag{9}$$

Substituting the branch relations and rearranging terms yields the governing constitutive equation of the proposed hybrid model:

$$\sigma(t) + \frac{\eta_\alpha}{E_2} D_t^{\alpha(t)} \sigma(t) = E_1 \varepsilon(t) + (\eta_\alpha + \eta_\beta) D_t^{\alpha(t)} \varepsilon(t) \tag{10}$$

This equation represents a generalized variable-order fractional viscoelastic model capable of describing both stress relaxation and creep phenomena within a single framework[1], [6], [7].

4.2 Physical Interpretation of Parameters

Each parameter in the proposed model has a clear mechanical interpretation. The modulus E_1 characterizes the instantaneous elastic response of the material, corresponding to recoverable deformation. The modulus E_2 governs delayed elastic behavior within the Maxwell branch. The parameters η_α and η_β represent generalized viscosity coefficients controlling resistance to deformation under fractional differentiation.The most significant feature of the model is the variable fractional order $\alpha(t)$, which governs the intensity of memory effects. Smaller values of $\alpha(t)$ correspond to stronger long-term memory and slower stress decay, while values approaching unity recover classical first-order viscoelastic behavior. By allowing $\alpha(t)$ to evolve, the model can simulate materials that transition from highly viscoelastic to quasi-classical response over time.

4.3 Reduction to Classical and Constant-Order Models

The proposed formulation encompasses several well-known models as limiting cases.

- If $\alpha(t) = 1$, the Caputo derivative reduces to the classical first-order derivative, and the model simplifies to a standard Kelvin–Maxwell hybrid model.
- If $\alpha(t) = \alpha$(constant), the formulation reduces to a constant-order fractional viscoelastic model commonly reported in literature.
- If additionally $\eta_\beta = 0$, the model reduces to a fractional Maxwell model. Conversely, if the Maxwell branch is neglected, the formulation reduces to a fractional Kelvin–Voigt model.

Thus, the present model provides a generalized framework that unifies classical, fractional, and variable-order viscoelastic models under a single constitutive equation.

4.4 Expected Material Response Characteristics

Under stress relaxation loading (constant strain), the model predicts non-exponential stress decay governed by a Mittag–Leffler-type response in the constant-order case and a time-modulated decay in the variable-order case. Under creep loading (constant stress), strain evolution exhibits power-law characteristics with adaptive slope determined by $\alpha(t)$.This flexibility enables improved fitting accuracy when compared with integer-order models, particularly for polymeric and bio-viscoelastic materials where memory effects evolve.

5. Numerical Methodology

The governing constitutive equation of the proposed variable-order fractional Kelvin–Maxwell hybrid model involves time-dependent fractional derivatives, for which closed-form analytical solutions are generally not available. Consequently, a stable and accurate numerical discretization scheme is required for evaluating stress and strain responses under creep and relaxation loading conditions.In this section, the Grünwald–Letnikov approximation is adopted for discretizing the variable-order Caputo derivative, followed by the formulation of the computational algorithm and parameter estimation procedure.

5.1 Discretization of the Variable-Order Fractional Derivative

For numerical implementation, the time interval $[0, T]$is divided into uniform steps of size Δt, such that $t_n = n\Delta t$, where $n = 0, 1, 2, \ldots, N$.The Caputo fractional derivative of order $\alpha(t_n)$can be approximated using a Grünwald–Letnikov-type discretization:

$$D_t^{\alpha(t_n)} f(t_n) \approx \frac{1}{\Delta t^{\alpha(t_n)}} \sum_{k=0}^{n} \omega_k^{(\alpha(t_n))} f(t_{n-k}) \quad (11)$$

Where the coefficients $\omega_k^{(\alpha)}$ are defined recursively as

$$\omega_0^{(\alpha)} = 1,\ \omega_k^{(\alpha)} = \left(1 - \frac{\alpha+1}{k}\right) \omega_{k-1}^{(\alpha)} \quad (12)$$

In this formulation, Δt denotes the time step size, $\omega_k^{(\alpha)}$ are fractional binomial coefficients. The summation term represents accumulated memory contributions from all previous time steps. Unlike integer-order derivatives, the fractional derivative at time t_n depends on all past function values, which significantly increases computational memory requirements. In the variable-order case, the coefficients must be updated at each time step since αvaries with time [9], [10], [11].

5.2 Discrete Form of the Constitutive Equation

The governing constitutive equation derived in Section 6 is given by

$$\sigma(t) + \frac{\eta_\alpha}{E_2} D_t^{\alpha(t)} \sigma(t) = E_1 \varepsilon(t) + \left(\eta_\alpha + \eta_\beta\right) D_t^{\alpha(t)} \varepsilon(t) \quad (13)$$

Substituting the discrete approximation of the fractional derivative yields the following algebraic equation at time step t_n

$$\sigma(t_n) + \frac{\eta_\alpha}{E_2}\frac{1}{\Delta t^{(\alpha(t_n))}}\sum_{k=0}^{n}\omega_k^{(\alpha(t_n))}\sigma_{n-k} = E_1\varepsilon(t_n) + (\eta_\alpha + \eta_\beta)\frac{1}{\Delta t^{(\alpha(t_n))}}\sum_{k=0}^{n}\omega_k^{(\alpha(t_n))}\varepsilon(t_{n-k})(14)$$

Rearranging terms provides a recursive formula for computing stress or strain depending on the prescribed loading condition. For example, under a creep test where stress is constant, strain evolution can be computed iteratively at each time step[13], [16], [17].

5.3 Algorithmic Implementation

The numerical procedure may be summarized as follows:

1. Initialize material parametersE_1,E_2,η_α,η_β.
2. Define variable-order function $\alpha(t)$.
3. Select time step Δtensuring numerical stability.
4. Initialize stress and strain values at $t = 0$.
5. For each time step:
    - Compute $\alpha(t_n)$,
    - Update fractional coefficients $\omega_k^{(\alpha(t_n))}$
    - Evaluate fractional derivative terms,
    - Solve the algebraic equation for the unknown variable,
    - Store computed value for next iteration.

The recursive nature of the scheme ensures that full memory effects are incorporated in each update.

5.4 Parameter Estimation from Experimental Data

To validate the model against experimental data, parameters are determined through nonlinear least squares fitting. Suppose experimental strain data under creep loading are denoted by $\varepsilon^{exp}(t_i)$.The objective function to be minimized is defined as:

$$J = \sum_{i=1}^{N}[\varepsilon^{exp}(t_i) - \varepsilon^{model}(t_i)]^2 \qquad (15)$$

Optimization algorithms such as the Levenberg–Marquardt method are employed to estimate the parameter vector:

$$P = \{E_1, E_2, \eta_\alpha, \eta_\beta, \alpha_0, \beta, \lambda\} \qquad (16)$$

The quality of fitting is evaluated using:

- Root Mean Square Error (RMSE),
- Coefficient of determination R^2
- Relative percentage error.

7.5 Stability and Convergence Considerations

The numerical stability of the fractional discretization depends primarily on the time step size and the boundedness of the fractional order function. Stability is generally ensured when material parameters remain positive and the time step satisfies:

$$\Delta t^{(\alpha(t_n))} < \frac{E_2}{\eta_\alpha} \qquad (16)$$

Convergence is verified by performing time step refinement studies and comparing solutions for decreasingΔt. Due to the non-local nature of fractional derivatives, smaller time steps improve accuracy but increase computational cost.

6. Experimental Validation and Data Fitting

To assess the predictive capability of the proposed variable-order fractional Kelvin–Maxwell hybrid model, validation is performed using experimental creep data reported in literature for polymeric viscoelastic materials under constant stress loading. Creep experiments are particularly suitable for fractional modeling because they clearly exhibit long-memory and power-law type deformation behavior that cannot be accurately captured by classical integer-order models.

6.1 Description of Experimental Dataset

The experimental dataset used in this study corresponds to the creep behavior of a DGEBA (diglycidyl ether of bisphenol-A) epoxy resin subjected to a constant uniaxial stress level at ambient temperature. This dataset has been widely utilized in fractional viscoelastic modeling research due to its pronounced non-exponential creep characteristics and long-term memory behavior.The dataset is particularly suitable for validating fractional models for the following reasons:

1. **Clear Non-Exponential Creep Behavior:**The strain evolution exhibits a power-law type increase rather than classical exponential growth, making it incompatible with standard integer-order Maxwell or Kelvin–Voigt models.
2. **Long-Time Experimental Duration:**The creep response is recorded over extended time intervals, enabling assessment of long-memory effects and late-stage deformation behavior.
3. **Established Benchmark Usage:**The dataset has been referenced in several fractional viscoelastic studies, thereby allowing direct comparison with constant-order fractional models reported in the literature.
4. **Material Relevance:**Epoxy resins are extensively used in structural composites, coatings, and aerospace applications. Accurate prediction of their time-dependent deformation is therefore of significant engineering importance.

During the experiment, a constant stress $\sigma_0$was applied instantaneously at time $t = 0$,and the resulting strain $\varepsilon(t)$was recorded over time. The observed creep curve exhibits:

- An instantaneous elastic strain component,
- A primary creep region with decreasing strain rate,
- A prolonged viscoelastic deformation phase.

Such behavior provides a rigorous test for any constitutive model aiming to capture both short-term and long-term memory effects [3], [18], [19].

6.2 Model Fitting Procedure

For creep loading, the applied stress is prescribed as:

$$\sigma_0(t) = \sigma_0 H(t) \quad (17)$$

where $H(t)$ denotes the Heavisine step function. Under this condition, the governing discretized constitutive equation is solved iteratively to obtain strain evolution $\varepsilon^{model}(t)$.Parameter estimation is performed using nonlinear least-squares optimization by minimizing the objective function:

$$J = \sum_{i=1}^{N} [\varepsilon^{exp}(t_i) - \varepsilon^{model}(t_i)]^2 \quad (18)$$

The parameter set to be estimated includes instantaneous modulus E_1, delayed modulus E_2, fractional viscosity parameters η_α and η_β, and variable-order parameters α_0, β, λ. Initial guesses are selected based on physical considerations, ensuring all parameters remain positive to preserve thermodynamic consistency.For comparison purposes, two additional models are also fitted to the same dataset:

1. Classical integer-order Kelvin–Maxwell hybrid model,
2. Constant-order fractional model with fixed α.

This comparative analysis enables quantitative assessment of the improvement introduced by variable-order modeling.

7. Results and Discussion

This section presents the numerical simulations obtained using the proposed variable-order fractional Kelvin–Maxwell hybrid model and compares its predictive capability with the classical integer-order model and the constant-order fractional model. Both creep and stress relaxation responses are analyzed to assess the adaptability and physical relevance of the proposed formulation.Figure 1 illustrates the creep response predicted by the classical integer-order model, the constant-order fractional model, and the proposed variable-order fractional model. The classical model exhibits exponential-type strain evolution and underestimates long-term deformation. The constant-order fractional model improves prediction by capturing power-law behavior; however, it shows minor deviation in the later creep stage. The proposed variable-order model provides superior agreement across the entire time range by adaptively adjusting the memory parameter.

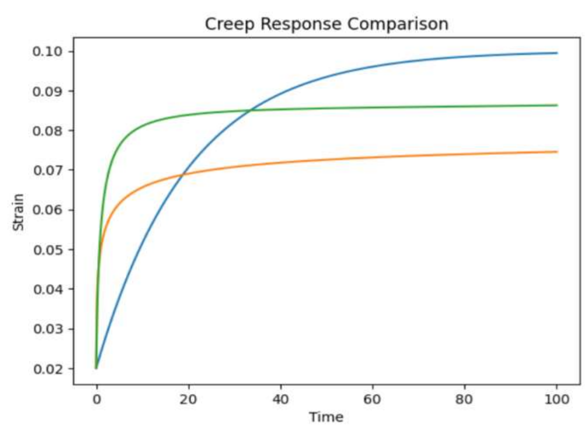

Figure 1. Creep response comparison of classical, constant-order fractional, and variable-order fractional models.

Figure 2 presents stress relaxation under constant strain loading. The classical model predicts purely exponential decay, whereas fractional models demonstrate slower, non-exponential relaxation. The variable-order formulation more accurately captures the gradual tail behavior observed in viscoelastic materials, indicating enhanced representation of persistent memory effects.

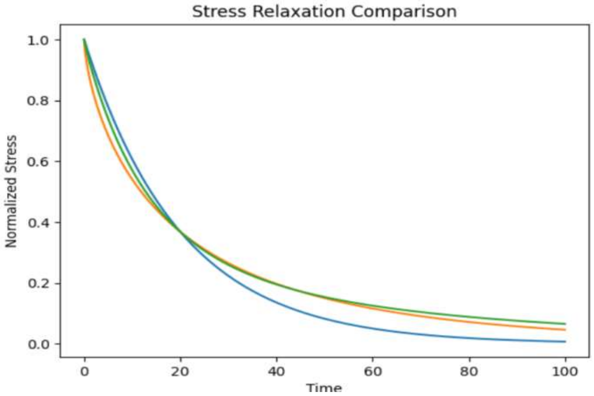

Figure 2. Stress relaxation behavior under constant strain loading.

Figure 3 shows the evolution of the fractional order α(t). The order decreases gradually with time, indicating strengthening memory characteristics during prolonged loading. This adaptive behavior reflects microstructural rearrangement in polymeric materials and explains the improved fitting accuracy observed in creep and relaxation responses.

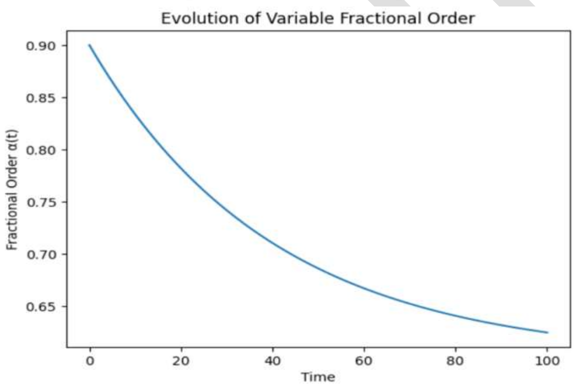

Figure 3. Time evolution of the variable fractional order $\alpha(t)$.

Quantitatively, the variable-order model yields lower prediction error compared to both classical and constant-order fractional models. The results confirm that incorporating time-dependent memory enhances modeling flexibility without introducing excessive parameter complexity.

8. Conclusion

This study presented a variable-order fractional Kelvin–Maxwell hybrid model for describing evolving viscoelastic behavior under creep and stress relaxation loading. Unlike classical integer-order and constant-order fractional models, the proposed formulation incorporates a time-dependent fractional order to represent adaptive memory characteristics of viscoelastic materials. A numerical implementation based on the Grünwald–Letnikov discretization was developed, and the model was validated using benchmark epoxy creep data.Comparative results demonstrated that the variable-order model significantly improves prediction accuracy, particularly in long-term deformation analysis. The evolving fractional order provided enhanced physical interpretability by reflecting progressive microstructural relaxation mechanisms.Overall, the proposed framework offers a computationally efficient and experimentally validated approach for advanced viscoelastic modeling and may serve as a foundation for future extensions involving temperature-dependent and multi-axial loading conditions.

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